II-2. On the Theory of the Ferrite Junction Circulator

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Among the several papers which have appeared in the literature pertaining to the ferrite junction circulator, those of Auld,[1] Bosma[2,3] and Butterweck[4] have seemed to us to be the most helpful in obtaining an understanding of the device. There have, however, been some points needing clarification, which we shall attempt to cover in this paper.

A phenomenological description of the operation of the stripline *Y*-junction circulator will be presented, followed by a mathematical justification and experimental evidence. The stripline *Y*-junction is the simplest geometrical form of the junction circulator and therefore the easiest to treat analytically. It is illustrated in Fig. 1, which shows a metallic center disk spaced from two conducting ground planes by means of two ferrite cylinders of the same diameter as the center disk. Connections to the center disk are by means of three stripline center conductors attached at points spaced 120° on its circumference. A dc magnetizing field is applied parallel to the axis of the ferrite cylinders.

It has been found experimentally that the *Y*-circulator has some, but not all, of the properties of a low-loss transmission cavity resonator. At its operating center frequency, it is well matched in spite of the obvious discontinuity where the striplines enter the ferrite-loaded disk. Also, a definite standing wave exists in the disk structure. This behavior suggests a resonance of the disk structure as being an essential feature of the operation of the circulator. The lowest frequency cavity mode resonance of this type of structure is the dipolar mode having E-field vectors perpendicular to the plane of the disk and rf H-field vectors parallel to the plane of the disk, as illustrated in Fig. 2(a). The rf H-lines in one ferrite cylinder curve over

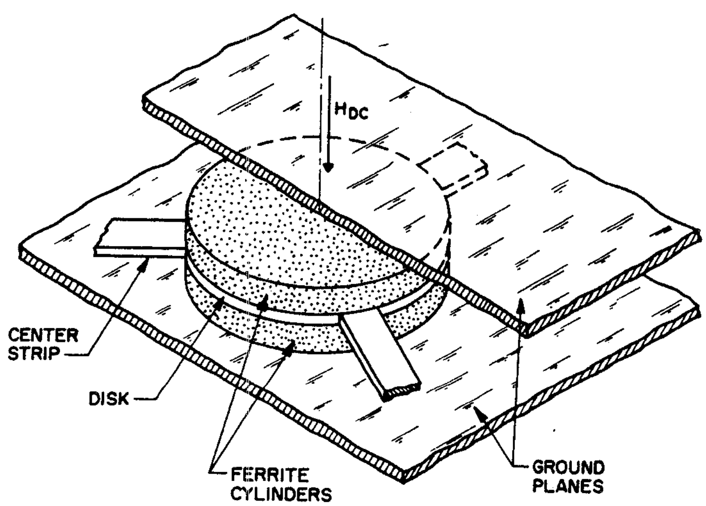

Fig. 1 Basic Y-junction circulator in stripline.

the edge of the disk and return in the other ferrite cylinder. Therefore, since the cylinders behave as mirror images, it is necessary to consider only one cylinder in the analysis.

If the device is excited at the input port and the ferrite is not magnetized, the standing wave pattern will be oriented as shown in Fig. 2 (a) when the other two ports are open-circuited. If the standing wave pattern is rotated as shown in Fig. 2 (b), then port 3 is situated at the null of the electric field, while ports 1 and 2 have equal field amplitudes. In this case, the device is equivalent to a transmission cavity between ports 1 and 2, and port 3 is isolated. The standing wave pattern of Fig. 2 (b) can be generated by two counter-rotating patterns of the same configuration. Each of these has an rf H-field which is circularly polarized at the center of the cylinder and linearly polarized at the edge. If a magnetic biasing field is applied parallel to the axis of the cylinders, the resonant frequencies of the two counter-rotating modes are different, i.e., the mode is "split" since the permeability in the center region will depend on whether the mode rotation is in the same sense or opposite to the sense of the spin precession in the ferrite. If the exciting frequency is between the two resonant frequencies of the split mode, then the one with the higher resonant frequency will have an in-

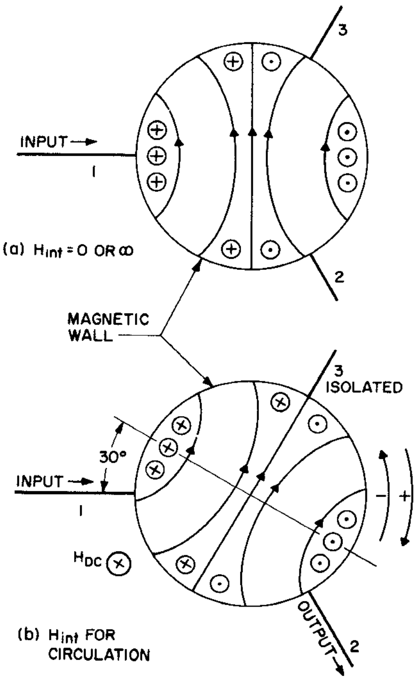

Fig. 2 Fields in the ferrite cylinder for the $n = 1$ mode. (a) With no magnetic biasing field. (b) With magnetic biasing field to create a 30° rotation of the field pattern.

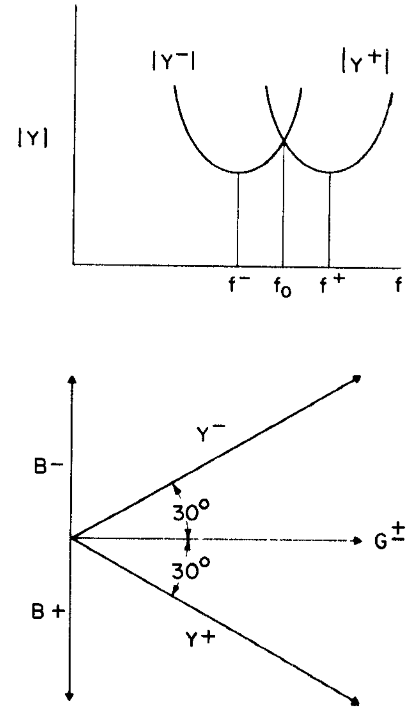

Fig. 3 The admittances of the split modes.

ductive susceptance (ratio of transverse magnetic-to-electric fields) at the exciting frequency, and the one with the lower resonant frequency will have a capacitive susceptance at the exciting frequency. If the exciting frequency is chosen so that the capacitive susceptance of one mode equals the inductive susceptance of the other mode, then the total admittance at the exciting frequency will be real. Further, if the amount of splitting of the modes is adjusted by means of the biasing field so that the phase angles of the two admittances are each 30° at the exciting frequency, as illustrated in Fig. 3, the standing wave pattern will be rotated 30° as shown in Fig. 2 (b). The "−" mode, or that whose sense of rotation is opposite to that of the electron spin precession, will have its current (or H-field) maximum at the input port 30° ahead of its voltage (or E-field) maximum, while the "+" mode will have its current maximum 30° behind its voltage maximum at the input port. Therefore, the two voltage maxima will concide 30° away from the input port as indicated in Fig. 2 (b).

The preceding physical picture can be justified analytically. It is found[2] that the rotating TE normal modes of the uncoupled ferrite cylinder with angular variation $e^{\pm jn\varphi}$ have resonant frequencies given by the roots of

$$J_{n-1}(kR) - \frac{nJ_n(kR)}{kR}\left(1 \pm \frac{\kappa}{\mu}\right) = 0,$$

where μ and κ are the on and off diagonal components of the Polder tensor, and the effective propagation constant is

$$k = \omega\sqrt{\left(\frac{\mu^2-\kappa^2}{\mu}\right)\mu_0\varepsilon\varepsilon_0} = \omega\sqrt{\mu_{eff}\mu_0\varepsilon\varepsilon_0}.$$

In the case of the three port circulator, the lowest operating frequency occurs between the resonances for $n = 1$. For this circulator in the lightly coupled case, it has been found experimentally that only the $n = 1$ mode need be considered as far as describing the electric field in the disk; however, for the magnetic field, the higher modes are necessary to satisfy the admittance boundary conditions at the strips and the magnetic wall boundary conditions away from the stripline connections. When this is done, it is found that the input admittance (assuming no z-directed electric or Φ-directed magnetic fields at the isolated port) can be expressed as $Y_{in} = Y^+ + Y^-$, where

$$Y^{\pm} = j\frac{Y_{eff}}{2J_1(kR)}\left[J_0(kR) - \frac{J_1(kR)}{kR}\left(1 \pm \frac{\kappa}{\mu}\right)\right]\left(1 \pm \frac{j}{\tan 120^\circ}\right),$$

with Y_{eff} given by $\sqrt{\varepsilon\varepsilon_0/\mu_0\mu_{eff}}$. From this expression we see that the phase angles of the rotating mode admittances are $\pm 30^\circ$, as described earlier. The frequency at which circulation obtains is also given by the theory and is found to be exactly given in the $n = 1$ mode case by the roots of

$$J_0(kR) - \frac{J_1(kR)}{kR} = 0$$

which lie between the "+" and "−" mode resonances. The lowest root is $kR = 1.84$. These equations can be used to guide the design of a stripline circulator, since the equations $Y_{in} = Y_0$ (stripline wave admittance) and $kR = 1.84$ involve $4\pi M_s$, H_0 and R, the cylinder radius.

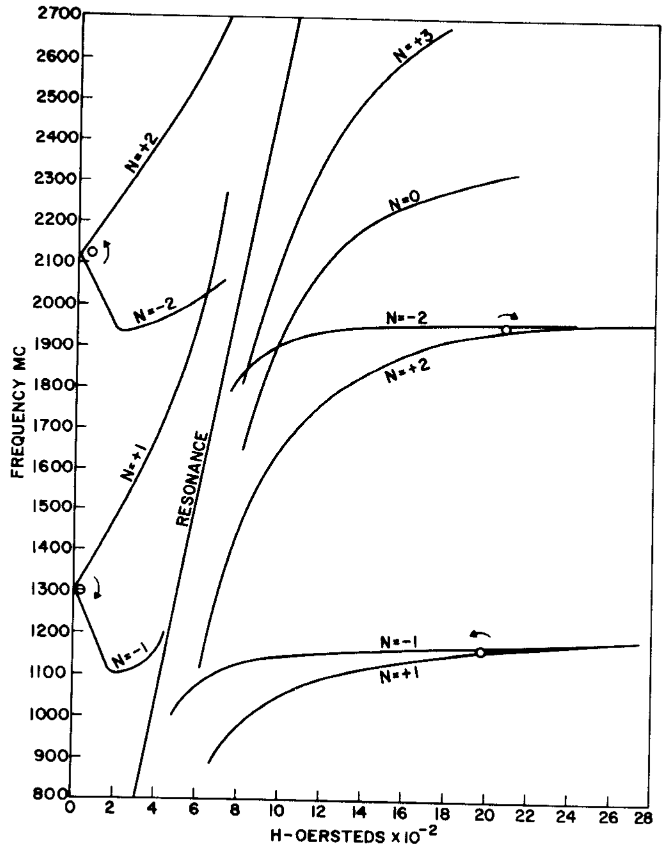

Fig. 4 Mode chart of a lightly coupled Y-circulator. Points and sense of circulation are indicated. H_{DC} vector points toward viewer.

In Fig. 4 we show a mode plot taken on a very lightly coupled *Y*-circulator. The various modes are labeled and their frequencies plotted as a function of the external biasing field. Points of circulation are indicated. It will be noted that below ferrimagnetic resonance, circulation is obtained at very low applied fields almost as soon as a splitting of the modes is observed.

Figure 5 shows a plot of the axial electric field taken on a lightly coupled circulator around the circumference of the center disk. The distribution is very nearly sinusoidal except near the stripline connections. Figure 6 shows a similar plot taken on a more heavily coupled circulator (~ 15% bandwidth at 25 db isolation). The principal differences from the lightly coupled case are the greater departure from sinusoidal distribution at the connecting striplines, and the shallower minimum between the input and output ports.

The waveguide *Y*-junction circulator can be treated in a manner similar to the foregoing example. A TM_{110} cavity containing a central ferrite post is connected to the waveguides by small irises to provide the lightly coupled

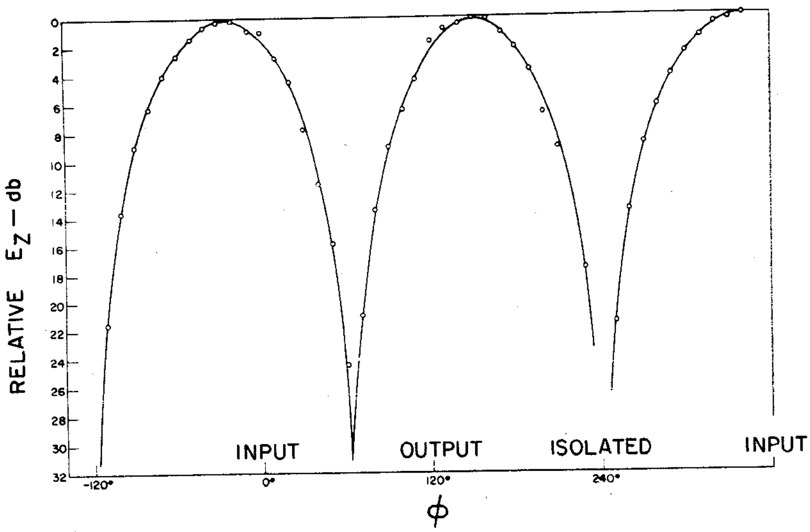

Fig. 5 Electric field distribution around circumference of the disk—lightly coupled case, $H_{DC} \approx 20$ Oersteds.

case. The more heavily coupled device is realized by enlarging the irises to the full waveguide cross section.

While the analysis given in this treatment is based on a circular geometry, the principles remain valid in other threefold symmetrical geometries which are sometimes used for *Y*-circulators. In particular, the concept of a

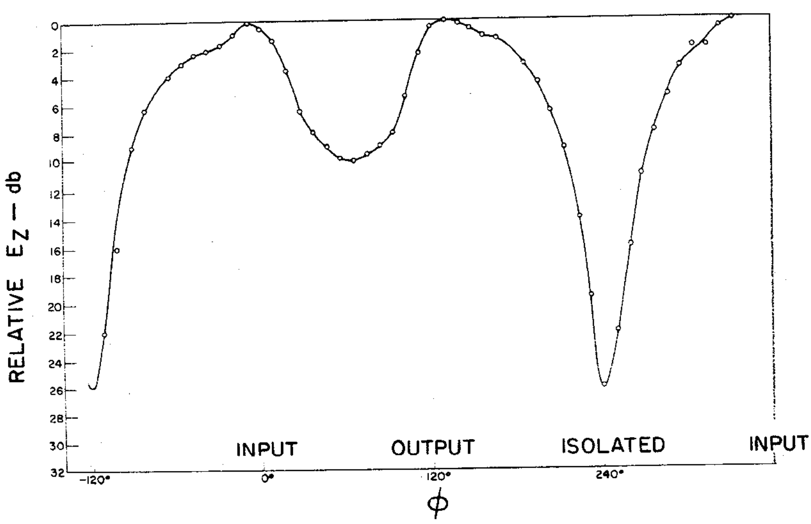

Fig. 6 Electric field distribution around circumference of the disk—heavily coupled case, $H_{DC} \approx 200$ Oersteds.

Y-circulator as a resonator whose standing wave pattern can be rotated to provide isolation of one port should be most useful in the treatment of this device.

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